11. Trace the curve $y^2 = (x-1)(x-2)(x-3)$. 15

12. (a) If $I_n = \int_0^{\pi/4} \tan^n x dx$; prove that $I_{n-1} + I_{n+1} = \frac{1}{n}$.

Deduce the value of I_5. 10

(b) Evaluate $\int \frac{xe^x}{(x+1)^2} dx$. 5

13. If $y = \sin(m \sin^{-1} x), |x| < 1$; prove that :

(a) $(1-x^2)y_2 - xy_1 + m^2 y = 0$ 5

(b) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$

and find value of $y_n(0)$. 10

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(21216) **Roll No. ....**

BCA-I Sem.

18005

B. C. A. Examination, Dec. 2016

MATHEMATICS-I

(BCA-101)

(New Course)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from all Sections as per instructions.

Section-A

(Very Short Answer Questions)

Attempt all the *five* questions of this Section. Each question carries 3 marks. Very short answer is required. 3×5=15

1. Show that $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is nilpotent matrix of order 2. 3

2. If $y = \log[\log(\log x)]$, find $\frac{dy}{dx}$. 3

3. Evaluate $\int \frac{(a^x - b^x)^2}{a^x b^x} dx.$ 3

4. Calculate the area of parallelogram spanned by the vectors $a = (3, -3, 1)$ and $b = (4, 9, 2)$. 3

5. Evaluate $\lim_{n \to \infty} \frac{\Sigma n^2}{n^3}$. 3

Section-B

(Short Answer Questions)

Attempt any *two* questions out of the following three questions. Each question carries 7½ marks. Short answer is required. 7½×2=15

6. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. 7½

7. Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)} dx.$ 7½

8. Expand $\sin x$ in power of x and hence find $\sin 18^\circ$ upto four decimal places. 7½

Section-C

(Detailed Answer Questions)

Attempt any *three* questions out of the following five questions. Each question carries 15 marks. Answer is required in detail. 15×3=45

9. (a) Show that $\lim_{x \to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$ does not exist. 5

(b) Examine the continuity of the function at the indicated point. Also point out the type of discontinuity; if any : 10

$$f(x) = \begin{cases} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \quad \text{at } x = 0.$$

10. Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

and hence find A^{-1}. Also state the Cayley-Hamilton theorem. 15

10. Examine the continuity of the function:

$$f(x) = \begin{cases} -x^2 & \text{if} \quad x \le 0 \\ 5x - 4 & \text{if} \quad 0 < x \le 1 \\ 4x^2 - 3x & \text{if} \quad 1 < x < 2 \\ 3x + 4 & \text{if} \quad x \ge 2 \end{cases}$$

at x=0, 1 & 2. 15

11. Trace the curve $x^3+y^3=3\,axy$. 15

12. (i) Expand log (1+x) by Maclaurin's Theorem. 15

(ii) Expand sinx in powers of $(x-\pi/2)$ by using Taylor's Theorem. 15

13. Evaluate :

(i) $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$

(ii) $\int \sin x \sin 2x \sin 3x \, dx$

N (Printed Pages 4)

(201217) Roll No......................

BCA-I Sem.

18005

B.C.A. Examination, Dec. 2017

MATHEMATICS-I

(BCA-101)

(New Course)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from **all** sections as per instructions.

Section-A

(Very Short Answer Questions)

Note : Attempt all the **five** questions of this section. Each question carries **3** marks. Very short answer is required. 3×5=15

1. Show that $A = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix}$ is Hermitian.

P.T.O.

2. Define continuity of a function at a point. 3

3. Show that the parabola $y^2-4ax=0$ has no asymptote. 3

4. Define Gamma and Beta function. 3

5. Define vector in 3-dimensions with example. 3

Section-B

(Short Answer Questions)

Note : Attempt any **two** questions out of the following **three** questions. Each question carries 7½ marks. Short answer is required. 7½×2=15

6. A function f(x) is defined as follows:

$$f(x) = \begin{cases} (x^2/a) - a, & \text{when } x < a \\ 0 & , \text{ when } x = a \\ a - (a^2/x), & \text{when } x > a \end{cases}$$

Prove that the function f(x) is continuous at x=a.

7. Find n^{th} differential coefficient of $x^3 \cos x$.

8. Find the area of a rectangle having vertices A,B,C & D with position vectors.

$$a = \left(-1, \frac{1}{2}, 4\right),\ b = \left(1, \frac{1}{2}, 4\right)$$

$$c = \left(1, \frac{-1}{2}, 4\right),\ d = \left(-1, \frac{-1}{2}, 4\right)$$

respectively.

Section-C

(Detailed Answer Questions)

Note : Answer any **three** questions out of the following **five** questions. Each question carries **15** marks. Answer is required in detail. 15×3=45

9. Verify Cayley-Hamilton theorem 15

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Also determine the characteristic roots and corresponding characteristic vector of the matrix A.

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10. Check the continuity of the following functions at $x=0$:

(i) $f(x)=\frac{|x|}{x}$ for $x \neq 0$ and $f(0)=0$

(ii) $f(x)=\frac{e^{1/x}\sin(1/x)}{1+e^{1/x}}$ for $x \neq 0$ and $f(0)=0$.

11. Trace the curve $9ay^2=(x-2a)(x-5a)^2$.

12. (i) Expand $\frac{e^x}{e^x+1}$ by Maclaurin's theorem.

(ii) Expand $\log x$ in power of $(x-1)$ by Taylor's theorem.

13. Evaluate :

(i) $\int \tan^4 x \, dx$

(ii) $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$.

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(21218) Roll No. ....

BCA-I Sem.

18005

B. C. A. Examination, Dec. 2018

MATHEMATICS-I

(BCA-101)

(New Course)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from all Sections as per instructions.

Section-A

(Very Short Answer Questions)

Attempt all the *five* questions. Each question carries 3 marks. Very short answer is required. 3×5=15

1. Show that $A=\begin{bmatrix} 3 & 1+2i \\ 1-2i & 2 \end{bmatrix}$ is Hermitian.

2. Define limit of a function at a point.

3. Find the asymptotes of the curve $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

4. State fundamental theorem of calculus.

5. Define vector in 2-dimension with example.

Section-B

(Short Answer Questions)

Attempt any *two* questions out of the following three questions. Each question carries 7½ marks. Short answer is required. 7½×2=15

6. Determine the values of a, b, c for which the function :

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & \text{for } x < 0 \\ c & \text{for } x = 0 \\ \dfrac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} & \text{for } x > 0 \end{cases}$$

is continuous at $x = 0$.

7. If $y = a\cos(\log x) + b\sin(\log x)$, show that :

$$x^2y_2 + xy_1 + y = 0 \text{ and } x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

8. Calculate the area of parallelogram spanned by the vectors $a = (1, -1, 3)$ and $b = (2, -7, 1)$.

Section-C

(Detailed Answer Questions)

Attempt any *three* questions out of the following five questions. Each question carries 15 marks. Answer is required in detail. 15×3=45

9. Verify Cayley -Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}.$$

Also determine the characteristic roots and corresponding characteristic vector of the matrix A.

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13. (i) Find the Rank of the matirx

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

(ii) Find the adjoint of the matirx

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

A **Printed Pages : 4**

(21119) **Roll No. ....**

BCA-I Sem.

18005

B.C.A. Examination, November 2019

MATHEMATICS-I

(BCA-101)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from **all** Sections as per instructions.

Section-A

Note : *Attempt all the five question of this section. Each question carries 3 marks. Veryshort answer is required.* 5×3=15

1. Define rank of a matrix.

2. Show that $\underset{x \to 2}{\text{limit}} \dfrac{|x-2|}{x-2}$ does not exist.

3. Verify Rolle's theorem for the function.

$f(x) = 2x^3 + x^2 - 4x - 2, \ x \in \left[-\sqrt{2}, \sqrt{2}\right]$

4. Evaluate :

$$\int x^2 e^x \, dx$$

5. Write the formula of $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b}$.

Section-B

Note : *Attempt any **two** questions out of the following three questions. Each question carries 7½ marks. Short answer is required.*

2×7½=15

6. Solve the following system of equations by Cramers Rule

$$3x + 4y = 5$$
$$x - y = -3$$

7. Differentiate $(\sin x)^x$

8. Evaluate :

$$\int \frac{xe^x}{(1+x)^2} \, dx$$

Section-C

Note : *Answer any **three** questions out of the following five questions. Each question carries 15 marks. Answer is required in detail.*

3×15=45

9. (i) Given :

$$\bar{a} = \hat{i} + 3\hat{j} - 2\hat{k}$$
$$\bar{b} = -\hat{i} + 3\hat{k}$$

Find $\bar{a} \cdot \bar{b}$ and $|\bar{a} \times \bar{b}|$

(ii) Find the unit vector perpendicular to both the vectors

$$4\hat{i} - \hat{j} + 3\hat{k} \text{ and } -2\hat{i} + \hat{j} - 2\hat{k}$$

10. Evaluate the following Integral of limit of sum

$$\int_a^b x \, dx.$$

11. Evaluate by L′ Hospital rule

(i) $\displaystyle \lim_{x \to 1} \frac{x^5 - 2x^3 - 4x^2 + 9x - 4}{x^4 - 2x^3 + 2x - 1}$

(ii) $\displaystyle \lim_{x \to 0} \frac{x - \sin x}{x^3}$

12. (i) Evaluate $\displaystyle \lim_{x \to 0} \frac{x - |x|}{x}$

(ii) Given $f(x) = \frac{|x|}{x}$, for $x \neq 0$

and $f(0) = 0$

show that $f(x)$ is not continuous at $x = 0$

 [P.T.O.

4. Give the statement of Rolle's theorem.

5. In short, explain Dot product and Cross product.

Section-B

Note : Attempt any **two** questions out of the three questions. Each question carries 7½ marks. 2×7½=15

6. Solve the following equations by Cramer's Rule

$3x+4y=5$

$x-y=-3$

7. Use Maclaurin's theorem to prove that

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\ldots(-1)^{n/2}\frac{x^n}{n!}+\ldots$$

8. If $I_n=\int_0^{\pi/3}\tan^n x\,dx$ then show that $(n-1)(I_n+I_{n-2})=(\sqrt{3})^{n-1}$

Section-C

Note : Attempt any **three** questions out of the following five questions. Each question carries 15 marks. 3×15=45

9. What do you mean by L-Hospital rule? Evaluate $\lim_{x\to\frac{\pi}{2}}\frac{\log\left(x-\frac{\pi}{2}\right)}{\tan x}$ by using L-Hospital Rule.

10. Examine the function f(x) given by $f(x)=10x^6-24x^5+15x^4-40x^3+108$ for maximum and minimum values.

11. If $\vec{F}=(x^2+y^2)\hat{i}-2xy\hat{j}$ and curve C is the rectangle in xy-plane bounded by $y=0$, $x=a$, $y=b$, $x=0$ then prove that

$$\oint_C \vec{F}.d\vec{r}=-2ab^2$$

11. (a) Differentiate $y = x \sin x \log x$.

(b) Find the maximum and minimum values of $(3x^4 - 8x^3 + 12x^2 - 48x + 25)$ in $[0, 3]$.

12. (a) Evaluate $\int x^2 \sin x \, dx$.

(b) Evaluate $\int (\sqrt{\sin x} \cdot \cos x) dx$.

13. (a) Show that the vectors

$\hat{i} - 3\hat{j} + 4\hat{k}, 2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - 7\hat{j} + 10\hat{k}$

are coplanar.

(b) Find the area of a parallelogram whose adjacent sides are determined by the vectors.

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -3\hat{i} - 2\hat{j} + \hat{k}$.

A (Printed Pages 4)

(20222) Roll No. ....

BCA-I Sem.

18005 (CV-III)

B.C.A. Examination, Dec.-2021

MATHEMATICS-I

(BCA-101)

Time : 1½ Hours] *[Maximum Marks : 75*

Note : Attempt questions from **all** sections as per instructions.

Section-A

(Very Short Answer Questions)

Note : Attempt any **two** questions of this Section. Each question carries **7.5** marks. Very short answer is required. 2×7.5=15

1. Define continuity at a point.

2. State Caley-Hamilton Theorem.

P.T.O.

function?

P.T.O.

3. If $y = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$, find $\frac{dy}{dx}$

4. Evaluate $\int \log_e x \, dx$.

5. Find λ such that $\vec{a}$ and $\vec{b}$ are perpendicular vector where

$\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}, \quad \vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$.

Section-B

(Short Answer Questions)

Note : Attempt any **one** question out of the following **three** questions. Each question carries **15** marks. Short answer is required. $1\times15=15$

6. Expand e^x in ascending powers of x by Maclaurin's theorem.

7. Differentiate x^x.

8. Prove that

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$

Section-C

(Detailed Answer Questions)

Note : Attempt any **two** questions out of the following **five** questions. Each question carries **22.5** marks. Answer is required in detail. $2\times22.5=45$

9. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

(b) Determine the eigen values of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

10. (a) Show that the function $f(x)=|x|$ is continuous at $x=0$.

(b) Evaluate

$$\lim_{h\to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)$$