12. (a) Find the shortest distance between the lines: 7½

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}.$$

(b) Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$. 7½

13. (a) Transform the equation $x^4\left(\frac{d^2y}{dx^2}\right)+a^2y=0$ by the substitution $x=\frac{1}{z}$. 7½

(b) If $f(x)=\log\left(\frac{1+x}{1-x}\right)$, show that : 7½

$$f(x)+f(y)=f\left(\frac{x+y}{1+xy}\right).$$

(20518) Roll No. ....

BCA- II Sem.

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B. C. A. Examination, May 2018

MATHEMATICS-II

(BCA-201)

(New)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from all Sections as per instructions.

Section-A

(Very Short Answer Questions)

Attempt all the *five* questions. Each question carries 3 marks. Very short answer is required.

3×5=15

1. Define the following with examples : 3

(i) Proper subset

(ii) Complement of a set

(iii) What is the set $\{x : x \in R,\ x^2=9,\ 2x=4\}$?

2. Let $f: A \to B$ such that $f(x) = x - 1$ and $g: B \to C$ such that $g(y) = y^2$. Find $fog(y)$. 3

3. Show that a linearly ordered poset is a distributive lattice. 3

4. If $u = \tan^{-1}\left(\frac{y}{x}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. 3

5. Evaluate $\int_0^{\pi/2}\int_0^{\sin\theta} r d\theta dr$. 3

Section-B

(Short Answer Questions)

Attempt any *two* questions out of the following three questions. Each question carries 7½ marks. Short answer is required. 7½×2=15

6. Show that the direction cosines of a line whose direction ratios are a, b, c are : 7½

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}.$$

7. In a group of 50 people, 35 speak Hindi, 25 speak both English and Hindi and all people speak at least one of the two languages. How many people speak only English and not Hindi? How many speak English? 7½

8. Show that $\sin x(1+\cos x)$ is a maximum at $x = \frac{\pi}{3}$. 7½

Section-C

(Detailed Answer Questions)

Attempt any *three* questions out of the following five questions. Each question carries 15 marks. Answer is required in detail. 15×3=45

9. Find the acute angle between two lines whose direction cosines are given by the relation $l+m+n=0$ and $l^2+m^2-n^2=0$. 15

10. Change the order of integration : 15

$$\int_0^a\int_x^{a^2/x} \phi(x, y) dx dy.$$

11. (a) Evaluate $\int_0^a\int_0^{a-x}\int_0^{a-x-y} x^2 dx dy dz$. 7½

(b) Evaluate $\int_0^1\int_0^{x^2} e^{y/x} dx dy$. 7½

(ii) If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$ then prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$

12. (i) Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar.

(ii) Find the angle of intersection of the spheres :

$$x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$$

and $x^2 + y^2 + z^2 - 6x - 2y + 2z + 2 = 0$

13. (i) Evaluate the double integral $\int_0^a \int_0^{\sqrt{(a^2-x^2)}} x^2 y\, dx\, dy$. Also mention the region of integration involved in this double integral.

(ii) Prove that the value of triple integration : $\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_0^{\sqrt{(1-x^2-y^2)}} xyz\, dz\, dy\, dx$, is $\frac{1}{48}$.

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Total Questions : 13] [Printed Pages : 4

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B.C.A. IInd Semester Examination, May-2019

MATHEMATICS-II

(BCA-201)

(New)

Time : 3 Hrs.] *[M.M. : 75*

Note :– Attempt all the Sections as per instructions.

Section–A

(Very Short Answer Type Questions)

Note :– Attempt all the *five* questions. Each question carries 3 marks.

1. Differentiate finite sets and infinite sets with example.
2. Define trigonometric function, exponential function and logarithmic function.

3. What do you mean by 'Principle of Duality' ?

4. If $u = f\left(\frac{y}{x}\right)$ then prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

5. Evaluate the triple integral $\int_0^1 \int_1^2 \int_2^3 dx\, dy\, dz$.

Section–B

(Short Answer Type Questions)

Note :– Attempt any *two* questions out of the following three questions. Each question carries 5 marks.

6. Define equivalence relation. If A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 3) (3, 3), (4, 4)}. Then prove that R is an equivalent relation.

7. Find the area of the region bounded by the circle $x^2 + y^2 = a^2$, by double integration.

8. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar. Also find their point of intersection.

Section–C

(Long Answer Type Questions)

Note :– Attempt any *three* questions out of the following five questions. Each question carries 15 marks.

9. (i) If Q be the set of rational numbers and $f : Q \rightarrow Q$ be defined by $f(x) = 2x + 3$ then prove that f is bijective. Also find f^{-1}.

(ii) If $f : R \rightarrow R$ and $g : R \rightarrow R$ be defined by $f(x) = x - 1$ and $g(x) = x^2 + 1$ then find $f \circ g(1)$, $f \circ g(2)$, $g \circ f(2)$, $f \circ f(2)$ and $g \circ g(2)$.

10. (i) Let (L, ≤) is a lattice. If $a, b \in L$ then prove that :

$$a \leq b \Leftrightarrow a \wedge b = a$$

and $$a \leq b \Leftrightarrow a \vee b = b$$

(ii) Let (L, ≤) be a lattice with least element 0 and greatest element 1. If $a \in L$ then show that :

$$a \vee 1 = 1 \text{ and } a \wedge 1 = a$$

Also $$a \vee 0 = a \text{ and } a \wedge 0 = 0$$

(i) Discuss the maxima or minima of the function :

$$u = xy + \left(\frac{a^3}{x}\right) + \left(\frac{a^3}{y}\right)$$

11. (a) If $x = r\cos\theta$, $y = r\sin\theta$, show that :

(i) $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1$

(ii) $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0.$

(b) Find the minima and maxima of $xy(a-x-y)$.

12. (a) Find the area between the line $y = x$ and curve $y = x^2$ enclosed in first quadrant.

(b) Evaluate by changing the order of integration :

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}.$$

13. (a) Find the equation of the tangent planes to the sphere $x^2+y^2+z^2=9$ which can be drawn through the line :

$$\frac{x-5}{2} = \frac{y-1}{2} = \frac{z-1}{1}.$$

(b) Find the equation of the line through the point (1, 2, 3) and parallel to the line

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(20516) Roll No. ....

BCA-II Sem.

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B. C. A. Examination, May 2016

MATHEMATICS–II

(BCA-201)

(New)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from all Sections as per instructions.

Section-A

(Very Short Answer Questions)

Attempt all the *five* questions of this Section. Each question carries 3 marks. $3\times5=15$

1. If A and B are two sets such that $A \cup B$ has 50 elements, A has 28 elements and B has 32 elements. How many elements does $A \cap B$ have?

2. Show that $\log_b a \times \log_c b \times \log_b c = 1$, where a, b, c all are positive numbers.

3. Find $\frac{\partial f}{\partial x}$, if $f = ye^{(x^2+y^2)}$.

4. Show that the planes $3x-2y+z+17=0$ and $4x+3y-6z-25=0$ are at right angles.

5. Evaluate $\int_0^3\int_1^2 xy(1+x+y)dx\,dy$.

Section-B

(Short Answer Questions)

This Section contains three questions, attempt any *two* questions. Each question carries 7½ marks.

7½×2=15

6. A function *f* from set of rational numbers to itself is defined by $f(x)=4x+3$. Show that f is a bijective function. Also find its inverse function.

7. If $u=\sin^{-1}\left\{\frac{x^2+y^2}{x+y}\right\}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$.

8. Find the distance of the point (0, 0, 0) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$.

Section-C

(Detailed Answer Questions)

This Section contains five questions, attempt any *three* questions. Each question carries 15 marks.

15×3=45

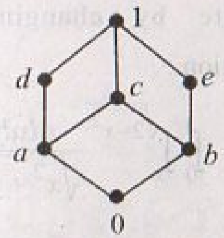
9. (a) Let $X=\{1,2,3\}$ and *f* and *g* be functions from X to itself given by $f=\{(1, 2), (2, 3), (3, 1)\}$ and $g=\{(1, 1), (2, 2), (3, 1)\}$. Find *fog* and *gof*.

   (b) Give examples of relations on the set $\{a,b,c\}$ which is :

   (i) reflexive but is neither symmetric nor transitive.

   (ii) Symmetric and transitive but not reflexive.

10. (a) Consider the lattice L in figure given below :



   (i) Is L a complemented lattice?

   (ii) Is L a complete lattice?

   (iii) Find complements, if they exist, for the elements a,b,c.

   (b) Show that the relation of divisibility is a partial order on the set N of natural numberse.

(ii) If $u = \log \frac{x^4 + y^4}{x + y}$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$$

12. (i) Find the equation to the plane passing through the four points (0, -1, -1), (4, 5,1), (3,9,4), (-4, 4, 4)

(ii) Find the equation of the sphere which passes through the points (1, -3, 4), (1, -5, 2), (1, -3, 0) and whose centre lies on the plane x + y + z = 0

13. (i) Evaluate the double integral

$$\int_{-a}^{a} \int_{\frac{-b}{a}\sqrt{a^2-x^2}}^{\frac{b}{a}\sqrt{a^2-x^2}} (x+y)^2 \, dx dy$$

(ii) Evaluate the triple integral

$\iiint(x^2+y^2+z^2)$ dx dy dz where R denotes the region bounded by x=0, y=0, z=0 and x+y+z =a, a>0

N (Printed Pages 4)

(20517) Roll No......................

BCA-II Sem.

18010

B.C.A. Examination, May 2017

MATHEMATICS - II

(BCA-201)

(New)

Time : Three Hours] *Maximum Marks : 75*

Note : Attempt questions from **all** Sections as per instructions.

Section-A

(Very Short Answer Questions)

Note : Attempt all the **five** questions of this Section. Each question carries 3 marks.

$3 \times 5 = 15$

1. Let A={2, 3, 5}, B={3, 6, 8} & C={4, 7, 9}. Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

2. Let Q be the set of rational numbers. Let f : $Q \to Q$ be defined by f(x) = 2x + 3. Show that f is bijective.

3. Show that the set of all factors of 12 under divisibility forms a lattice.

4. If $U = f(y/x)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

5. Find the direction cosines of the line segment joining the points P(2, 3, -6) and Q(3, -4, 5)

Section-B

(Short Answer Questions)

Note : This section contains **three** questions, attempt any **two** questions. Each question carries 7½ marks. 7½×2=15

6. Let Z be the set of integers, Define a relation R on I such that xRy if and only if x-y is divisible by $5 \forall x, y \in z$. Show that R is an equivalence relation.

7. Evaluate $\iint r^3 dr d\theta$ over the area bounded between the circles $r=2\cos\theta$ & $r=4\cos\theta$

8. Change the independent variable x to z in the equation $(1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + y = x$ by the substitution $x = \tan z$

Section-C

(Detailed Answer Questions)

Note : This section contains five questions, attempt any **three** questions. Each question carries 15 marks. 15×3=45

9. (i) Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = x - 1$ and $g(x) = x^2 + 1$. Find fog (2), gof(2), fof(2) and gog (2).

   (ii) If R & S be equivalence relations in the set X, then prove that $R \cap S$ is an equivalence relation in X.

10. (i) Let $(L, \leq)$ be a lattice and $a, b, c, d \in L$. Then show that

    (i) $(a \wedge b) \vee (c \wedge d) \leq (a \vee c) \wedge (b \vee d)$

    (ii) $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$

    (ii) Show that dual of a complemented lattice is complemented.

11. (i) If $V = f(x-y, y-z, z-x)$, then prove that $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0$

P.T.O.

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