12. (a) Solve :

$$\frac{d^2y}{dx^2} + 9y = \cos 2x + \sin 2x$$

(b) Solve :

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-3x}$$

13. (a) Solve :

$$(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$$

(b) Solve :

$$(x + 2y^3)\frac{dy}{dx} = y$$

(20519) Roll No. ....

Total Questions : **13**] [Printed Pages : **4**

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B.C.A. IVth Semester Examination, May-2019

MATHEMATICS–III

(BCA–406)

Time : 3 Hrs.] ***[M.M. : 75***

Note :– Attempt all the Sections as per instructions.

Section–A

(Very Short Answer Type Questions)

Note :– Attempt all the *five* questions. Each question carries 3 marks.

1. Find the order and degree of the differential equation :

$$\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^4 + y = 0$$

2. State Leibnitz test.

Or

Cauchy's root test.

3. Express $1 - i$ in the modulus amplitude form.

4. If $r = \sin t\,\hat{i} + \cos t\,\hat{j} + \hat{k}$, find :

(i) $\dfrac{dr}{dt}$

(ii) $\left|\dfrac{dr}{dt}\right|$

5. Solve :

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$$

Section–B

(Short Answer Type Questions)

Note :– Attempt any *two* questions out of the following three questions. Each question carries 7½ marks.

6. If z_1 and z_2 are any complex numbers, then prove that :

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\{|z_1|^2 + |z_2|^2\}$$

7. Solve :

$$(1 + x^2)dy = (1 + y^2)dx$$

8. Prove that :

$$\lim \frac{1}{n}\left(1 + 2^{1/2} + 3^{1/3} + \ldots\ldots + n^{1/n}\right) = 1$$

Section–C

(Long Answer Type Questions)

Note :– Attempt any *three* questions out of the following five questions. Each question carries 15 marks.

9. (a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then prove that :

$$x + y + z = xyz$$

(b) Test the convergence of the series whose nth term is :

$$\sqrt{n+1} - \sqrt{n}$$

10. Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.

11. Find the Fourier series for the function $f(x) = |x|$, $-\pi < x < \pi$. Hence deduce that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots$$

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